Solid Figures and Nets

Polyhedrons

Prisms







Pyramids







Not Polyhedrons







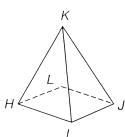
Cylinder

Cone

Sphere

Properties of polyhedrons include vertices, edges, and faces, and base(s).

Square Pyramid



Vertices: H, I, J, K, L

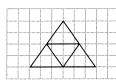
Edges: \overline{HI} , \overline{IJ} , \overline{JL} , \overline{LH} , \overline{HK} , \overline{IK} , \overline{JK} , \overline{LK} **Faces:** ΔHIK , ΔIJK , ΔJKL , and ΔHLK

Base: □*HIJL*

Nets Identify solid figures from a net: a pattern that folds into the solid.



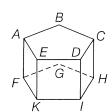
Cube



Pyramid

Classify the polyhedron. Name all vertices, edges, faces, and bases.

1.



Pentagonal prism; Vertices: A, B, C, D, E,

F, G, H, I, K

Edges: AB, AF, BC, BG, CD, CH, DE, DI,

EA, EK, FG, GH, HI, IK, KF

Faces: ABGF, BCHG, CDIH, DEKI,

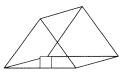
4.

4*EKF*

``ABCDE, (`)FGHIK Bases: (

In 2 through 4, classify each figure.

2.



Trianqular Drism

3.



Gylinder



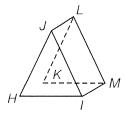


Bube

Solid Figures and Nets

Classify the polyhedron. Name all vertices, edges, faces, and bases.

1.



Triangular prism

Vertices: H, I, J, K, L, M

Edges: HI, HJ, JI, KL,

KM, LM, HK, JL, IM

Faces: JIML, JHKLJ, JHIMK

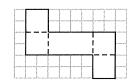
Bases: ΔHIJ , ΔKLM

Classify each figure.

2.

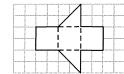


3.



Cube

4.



Triangular prism

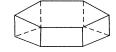
- 5. Which solid figure looks like a round cake? Cylinder
- **6. Number Sense** How many faces make up six number cubes?

36 faces

7. Reasoning A factory buys the boxes it needs in the form of flat nets. What advantages might the factory have in doing this?

Sample answer: Nets take up much less space than boxes and they can be folded into boxes when needed.

- 8. What is the name of the polyhedron shown below?
 - A Rectangular prism
 - B Hexagonal prism
 - C Pentagonal prism
 - **D** Octagonal prism



9. Writing to Explain Describe the similarities and differences of a cylinder and a cone.

A cylinder and a cone are solid figures, but they are not polyhedrons. They both have a curved surface, and a base that is circular. The cylinder has two circular bases and the cone has one.

Color Combinations

Ross has a 12-sided object, and each side is a pentagon. He wants to color it with only four colors: red, blue, green, and yellow.

The diagrams show the flattened views of the 12 faces. No faces with the same color should be next to each other. Finish the coloring for various combinations.

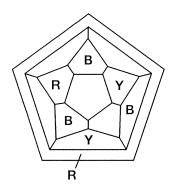


Visual Thinking

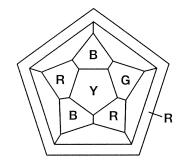
Check students' work.

Like colors may not touch.

1.



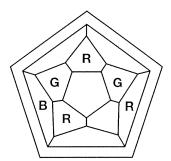
2.



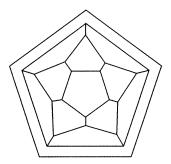
3.



4.



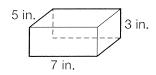
5. Design one of your own.



Surface Area of Prisms and Pyramids

You can use formulas to find the surface area of different solid figures. You can draw nets to help.

Rectangular Prism

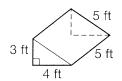


$$SA = 2\ell w + 2\ell h + 2wh$$

= 2 (5 × 7) + 2 (5 × 3) + 2 (7 × 3)
= 70 + 30 + 42
= 142

The surface area is 142 in².

Triangular Prism



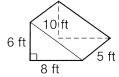
$$SA = 2 \left(\frac{1}{2} \times 4 \times 3\right) + (3 \times 5) + (4 \times 5) + (5 \times 5)$$

= 12 + 15 + 20 + 25
= 72

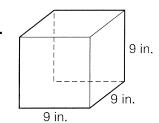
The surface area is 72 ft².

Find the surface area of each figure.

1.



2.



 $168 \, \text{ft}^2$

486 in²

Find the surface area of each rectangular prism.

3.
$$\ell = 5.5$$
 cm, $w = 4.5$ cm, $h = 3.5$ cm

4.
$$\ell = 15$$
 in., $w = 9$ in., $h = 3.8$ in.

5.
$$\ell = 2$$
 yd, $w = 6$ yd, $h = 1.7$ yd

$$119.5 \text{ cm}^2$$

$$51.2 \text{ yd}^2$$

6. Reasoning Write the dimensions of two different rectangular prisms that have the same surface area.

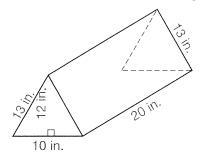
Sample answer: 2 by 2 by 2 and 1 by 3 by

2.25. Both rectangular prisms have a surface area of 24.

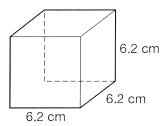
Surface Area of Prisms and Pyramids

Find the surface area of each figure. You can draw nets to help.

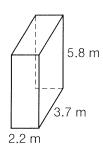
1.



2.



3.



840 in²

230.64 cm²

84.72 m²

Find the surface area of each rectangular prism.

4.
$$\ell = 6.9 \text{ mm}, w = 8.2 \text{ mm}, h = 14 \text{ mm}$$

5.
$$\ell = 3.4$$
 cm, $w = 12.7$ cm, $h = 16.5$ cm

6.
$$\ell = 5.7$$
 yd, $w = 9$ yd, $h = 12.9$ yd

535.96 mm²

617.66 cm²

481.86 yd²

7. Reasoning Margaret wants to cover a footrest in the shape of a rectangular prism with cotton fabric. The footrest is 18 in. \times 12 in. \times 10 in. She has 1 yd² of fabric. Can she completely cover the footrest?

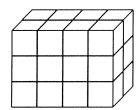
Yes; Sample answer: The footrest has an area of 1,032 in² and 1 yd² of fabric is 1,296 in².

- **8.** Which is the surface area of a rectangular prism with a length of 2.3 in., a width of 1.1 in., and a height of 3 in.?
 - **A** 26.48 in²
- **(B)** 25.46 in²
- **C** 24.58 in²
- **D** 21.5 in^2
- 9. Writing to Explain A square pyramid has 2 m sides on the base. Each face is a triangle with a base of 2 m and a height of 1.5 m. Explain how to find the surface area.

Sample answer:

Find the area of the base and the area of each triangular face. Multiply the area of the triangular face by 4 and add it to the area of the base. $4 + (4 \times 1.5) = 10 \text{ m}^2$

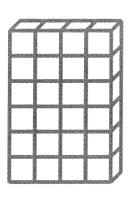
Block It!



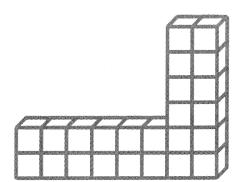
Visual Thinking

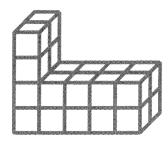
Draw six different arrangements you could make using all of the blocks in the solid figure shown.

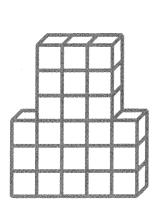
Sample answers:

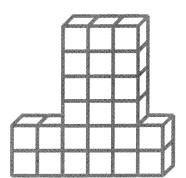












Modeling Volume

Volume is the measure of space inside a solid figure. It is measured in cubic units. You can use a formula to find the volume of rectangular prisms: $V = B \times h$ where V stands for volume, B stands for the area of the base, and h stands for the height.

To find the volume of the rectangular prism at the right, first find the area of the base.

$$B = \ell \times w$$

$$=4\times8$$

$$= 32$$

So the base is 32 sq in.

Then use the volume formula to find the volume.

$$V = B \times h$$

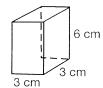
$$=32\times5$$

$$= 160$$

So the volume is 160 sq in.

Find the volume of each rectangular prism. Don't forget to label the units.

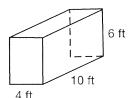
1.



Area of Base ($B = \ell \times w$): 9 cm²

Volume ($V = B \times h$): 54 cm³

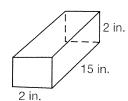
3.



Area of Base ($B = \ell \times w$):40 sq ft

Volume ($V = B \times h$): **240 cu ft**

2.



4 in.

Area of Base ($B = \ell \times w$): 30 sq in.

Volume ($V = B \times h$): **60 cu in.**

4.

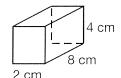


Area of Base ($B = \ell \times w$): 10 m^2

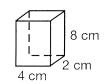
Volume ($V = B \times h$): 90 m³

5. Find the volume of Rectangular Prism 1. How can you find the volume of Rectangular Prism 2 without using the volume formula?

64 cm³; Sample answer: Rectangular Prism 2 is also



Rectangular Prism 1



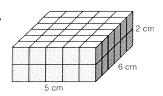
Rectangular Prism 2

64 cm³ because the same three values are used to measure the three dimensions. It doesn't matter in which order the numbers are multiplied—the product will be the same.

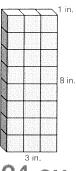
Modeling Volume

Find the volume of each rectangular prism.

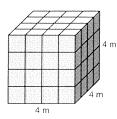
1.



2.



3.



60 cm³

24 cu in.

64 m³

Find the missing value for each rectangular prism.

7. Number Sense Suppose a box has a volume of 1 cu yd. What is its volume in cubic feet? 27 CU ft

- **8.** A rectangular prism has a base of 12 cm², a length of 3 cm, a width of 4 cm, and a height of 10 cm. Which is the volume of the prism?
 - **A** 36 cm³
 - **B** 48 cm³
 - (C) 120 cm³
 - **D** $1,440 \text{ cm}^3$
- **9. Writing to Explain** Find and compare the volumes of the two rectangular prisms below. How does doubling the measure of each dimension in a rectangular prism change the volume of the prism?

	Length	Width	Height	Volume
Rectangular Prism 1	5 ft	2 ft	10 ft	100 cu ft
Rectangular Prism 2	10 ft	4 ft	20 ft	800 cu ft

100 cu ft; 800 cu ft; The volume is 8 times the original volume.

Eye See

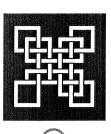
All the figures in each line are the same except for one. Circle the one that is different.

Visual Thinking

1.



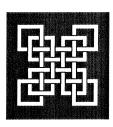
Α



(B)



С

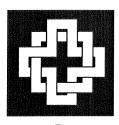


D



E

2.



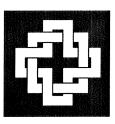
A



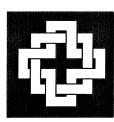
В



С



D



E

3.



Α



В



(C)



D



Ε

4.



Α



В



С



D



(E

5.



Α



В



C



D



E

Volume with Fractional Edge Lengths

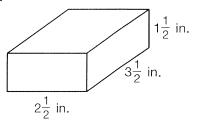
When finding the volume of a rectangular prism with fractional edge lengths, you have to find the number of cubes with fractional edge lengths that can fill the prism. What is the volume of the rectangular prism shown below at the right?

Consider a $\frac{1}{2}$ -inch cube. 8 half-inch cubes can fill a 1-inch cube.

Next, figure out how many $\frac{1}{2}$ -inch cubes will fill the prism. The prism can be filled with $5 \times 7 \times 3 = 105$ half-inch cubes.

Divide 105 by 8 because 8 half-inch cubes make up a 1-inch cube. $105 \div 8 = 13\frac{1}{8}$

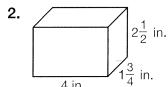
The volume of this rectangular prism is $13\frac{1}{8}$ in³.



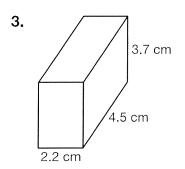
For 1 through 4, find the volume of each rectangular prism.

1.
$$10 \text{ in}$$
 $3\frac{1}{2} \text{ in}$ $3\frac{1}{2} \text{ in}$.

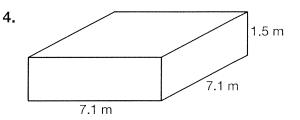
Fraction cubes have $\frac{1}{2}$ in. lengths. $V = \frac{52\frac{1}{2} \text{ in}^3}{2}$



Fraction cubes have $\frac{1}{4}$ in. lengths. $V = \frac{17\frac{1}{2} \text{ in}^3}{1}$



 $V = 36.63 \text{ cm}^3$



 $V = 75.615 \text{ m}^3$

5. Writing to Explain How many $\frac{1}{2}$ -inch cubes could fit inside the rectangular prism shown in Exercise 1? Explain how you know.

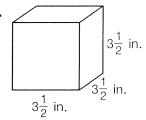
420 cubes; The dimensions of the prism in

 $\frac{1}{2}$ -inch cubes are $20 \times 7 \times 3 = 420$

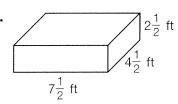
Volume with Fractional Edge Lengths

Find the volume of each rectangular prism.

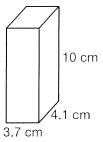
1.



2



3.



 $\frac{42\frac{7}{8} \text{ in}^3}{}$

84³/₈ ft³

151.7 cm³

Find the missing value for each rectangular prism.

4. Volume: $111\frac{3}{8}$ in³

Base: $20\frac{1}{4} \text{ in}^2$ Height: $5\frac{1}{2} \text{ in}$. 5. Volume: $8\frac{2}{3}$ ft³ Length: 3 ft

Width: $4\frac{1}{3}$ ft

Height: $\frac{2}{3}$ ft

6. Volume: 758.16 mm³

Length: 13 mm

Width: **8.1 mm**

Height: 7.2 mm

7. Number Sense A rectangular prism can be filled with 210 half-inch cubes. How many $\frac{1}{4}$ -inch cubes would it take to fill the same prism?

1,680 cubes

8. A rectangular prism has a base with an area of 31.5 cm² and a height of 4.7 cm. What is the volume of the prism?

A 36.2 cm³

(C) 148.05 cm³

B 72.4 cm^3

D 296.1 cm³

9. Writing to Explain Find and compare the volumes of the two rectangular prisms below. How does dividing each dimension of the larger prism by 2 affect the volume of the smaller prism?

Length	Width	Height	Volume
5 in.	$4\frac{1}{2}$ in.	6 in.	135 in ³
$2\frac{1}{2}$ in.	$2\frac{1}{4}$ in.	3 in.	$16\frac{7}{8}$ in ³

The volume of the smaller prism is $\frac{1}{8}$ the volume of the larger prism.

Backward Thinking

1. Mychala multiplied by 4 when she meant to subtract 4. Then she added 7 when she meant to multiply by 7. The answer she got was 39. What answer should Mychala have gotten?

Number Sense

28

2. In solving an addition problem, Rahim added 46,085 when he meant to add 46.85. In order to obtain the correct answer in a single step, what number should he subtract?

46.038.15

3. Jillian brought home a poster from her trip. She paid \$25 for the poster. Liu saw the poster and gave Jillian \$35 for it. A few days later, Jillian bought the poster back for \$45. Then she sold it again, this time to her cousin Allie for \$55. Did Jillian make money or lose money on her transactions? By how much?

made \$20

4. Victoria increased a number by 19 and then multiplied that result by 5, giving her an answer of 280. What was her original number?

37

5. Find a seven-digit number that fits these conditions.

4,333,336

- The number is a multiple of 7.
- Five of the number's digits are 3s and the other two are 4 and 6.
- The first digit is a 4.
- 6. Sinda purchased three blouses and two scarves for \$87.75. The two scarves were the same price, but one of the blouses cost \$5.00 more than the others. Five scarves would have cost \$64.75. What was the cost of the more expensive blouse?

\$23.95

7. Two factors, each without any digit greater than 5, have a product of 16,848. What is the largest possible three-digit factor with these conditions?

 $324 \times 52 = 16,848$

Problem Solving: Use Objects and Reasoning

Each cube has a volume of 1 cm³.

The area of one face of the cube is 1 cm².

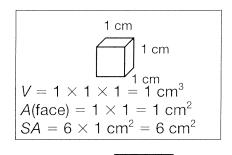
The surface area of the cube is the sum of the area of each face of the cube.

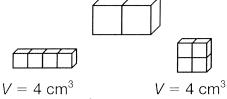
To find the surface area of a figure of cubes, count only the faces that are exposed.

$$V = 2(1 \times 1 \times 1) = 2 \text{ cm}^3$$

 $SA = 10(1 \text{ cm}^2) = 10 \text{ cm}^2$

The arrangement of cubes can affect the surface area, but the same number of cubes will always have the same volume.





$$V = 4 \text{ cm}^3$$

 $SA = 18 \text{ cm}^2$

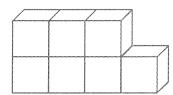
$$V = 4 \text{ cm}^3$$

 $SA = 16 \text{ cm}^2$

1. Find the volume and surface area of the figure.

$$V = 6 \text{ cm}^3$$
; $SA = 24 \text{ cm}^2$

2. Make a figure of cubes that has a volume of 7 cm³ and a surface area of 26 cm². Draw your figure.



3. Reasoning Explain how you know how many cubes to use to make the figure in problem 2.

The volume of each cube is 1 cm³, so 7 cm³ is equal to 7 cubes.

4. Find the volume and surface area of the figure.





5. Geometry If the cubes in problem 4 were increased to 3 cm on a side, how would the volume and surface area be affected?

The volume would be changed by a factor of $3 \times 3 \times 3$, or 27. The surface area would be changed by a factor of 3×3 , or 9.

Problem Solving: Use Objects and Reasoning

Find the volume and surface area of each figure of centimeter cubes.

1.
$$V = 5 \text{ cm}^3$$
;
 $SA = 22 \text{ cm}^2$

$$V = 5 \text{ cm}^3$$
;
 $SA = 20 \text{ cm}^2$

$$V = 6 \text{ cm}^3$$
;
 $SA = 22 \text{ cm}^2$

$$V = 6 \text{ cm}^3$$
;
 $SA = 24 \text{ cm}^2$

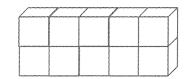
$$V = 6 \text{ cm}^3$$
;
 $SA = 26 \text{ cm}^2$

$$V = 8 \text{ cm}^3$$
;
 $SA = 30 \text{ cm}^2$

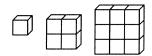
- 7. Make a figure of cubes that has a volume of 6 cm³ and a surface area of 22 cm². Draw your figure.
- 8. Critical Thinking Without building a model, tell whether a long row of 8 cubes or a cube made from 8 cubes would have a greater surface area. Explain.

A long row of cubes would expose more sides, so it would have a greater surface area.

9. Make a figure that has the same volume as the diagram, but a greater surface area. Draw your figure.



10. Writing to Explain Find the volume and surface area of these figures. Then describe the pattern(s) you see. Can you determine the volume of the next element in the pattern? The surface area? Explain.

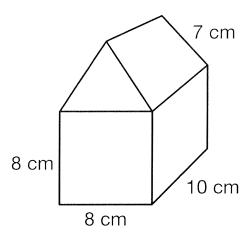


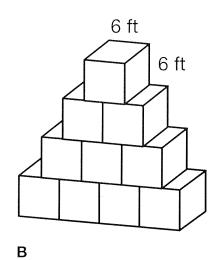
Volume	*	4	9
Surface Area	6	16	30

There is a pattern in the volumes of the figures: the volumes are the squares of the number of cubes on each side. The only pattern I can find in the surface areas is that they are even numbers.

What Should It Be?

Decision Making





Α

1. Herbert designed a model of a house shown in picture A. What is the surface area of the four walls of the model house? 288 cm²

2. Choose a design for some windows for the model house. Figure out how much surface area you should deduct or add.

Sample answer: 10 total windows, 1 cm² on each side, so subtract 10 cm²

3. The staircase design in picture B is made of cubes and is part of the set for a school play. The director wants to know how much room the staircase will use up in the school's storage locker. Find the volume of the staircase.

2,160 ft³

4. All parts of the stairs that can be seen need to be covered in silver wrap. How much wrap do you need?

1,152 sq ft

5. The play director asks you to redesign the staircase using the same number of cubes. Redo the design and figure how much wrap you need.

Sample answer:

1 row of 4 on top of 1 row of 6,

1,080 sq ft of wrap